Federal State Autonomous Educational Institution of Higher Education "Moscow Institute of Physics and Technology (National Research University)"

APPROVED
Vice Rector for Academic Affairs

A.A. Voronov

Work program of the course (training module)

course:	Linear Algebra/Линейная алгебра
major:	Biotechnology
specialization:	Biomedical Engineering/Биомедицинская инженерия
	Phystech School of Biological and Medical Physics
	Chair of Higher Mathematics
term:	2
qualification:	Bachelor

Semester, form of interim assessment: 3 (fall) - Exam

Academic hours: 60 AH in total, including:
- lectures: 30 AH.
- seminars: 30 AH.
- laboratory practical: 0 AH.

Independent work: 90 AH.

Exam preparation: 30 AH.

In total: 180 AH, credits in total: 4

Authors of the program:

D.V. Beklemishev, doctor of pedagogical sciences, full professor, professor

A.N. Burmistrov, candidate of physics and mathematical sciences, associate professor, associate professor

P.A. Kozhevnikov, candidate of physics and mathematical sciences, associate professor, associate professor

O.K. Podlipskiy, candidate of physics and mathematical sciences, associate professor, associate professor

I.A. Chubarov, candidate of physics and mathematical sciences, associate professor, associate professor

O.G. Podlipskaya, candidate of physics and mathematical sciences, assistant

The program was discussed at the Chair of Higher Mathematics 21.05.2020

Annotation

The discipline belongs to the basic part of the educational program. The development of the discipline is aimed at developing the ability to acquire new scientific and professional knowledge using modern educational and information technologies. Topics such as Matrices and systems of linear equations, Linear space, Linear dependencies in linear space, Nonlinear dependencies in linear space, Euclidean space, and Unitary space are considered.

1. Study objective

Purpose of the course

familiarization of students with the basics of linear algebra and preparation for the study of other mathematical courses – differential equations, the theory of functions of complex variables, equations of mathematical physics, functional analysis, analytical mechanics, theoretical physics, methods of optimal control, etc.

Tasks of the course

- students acquire theoretical knowledge and practical skills in the field of matrix algebra, the theory of linear spaces;
- preparing students for the study of related mathematical disciplines;
- acquisition of skills in the application of analytical methods in physics and other natural sciences.

2. List of the planned results of the course (training module), correlated with the planned results of the mastering the educational program

Mastering the discipline is aimed at the formation of the following competencies:

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically assess, and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

3. List of the planned results of the course (training module)

As a result of studying the course the student should:

know:

- operations with matrices, systems of linear algebraic equations matrices and determinants;
- theorems on systems of linear Kronecker-Capelli and Fredholm equations, Kramer rule, General solution of a system of linear equations;
- basic definitions and theorems on linear spaces and subspaces, on linear maps of linear spaces;
- definitions and basic properties of eigenvectors, eigenvalues, characteristic polynomial;
- reduction of the quadratic form to the canonical form, the law of inertia, the Sylvester criterion;
- coordinate recording of the scalar product, the basic properties of self-adjoint transformations;
- fundamentals of the theory of linear spaces in the volume that provides the study of analytical mechanics, theoretical physics and optimal control methods.

be able to:

- to produce a matrix, finding inverse of a matrix, to compute determinants;
- find a numerical solution to a system of linear equations. find eigenvalues and eigenvectors of linear transformations, bring the quadratic form to the canonical form, find the orthonormal basis of the eigenvectors of the self-adjoint transformation;
- operate with elements and concepts of linear space, including the main types of dependencies: linear operators, bilinear and quadratic forms.

master:

- general concepts and definitions related to matrix algebra;
- geometric interpretation of systems of linear equations and their solutions;
- concepts of linear space, matrix notation of subspaces and maps;
- conduct about the use of spectral problems;
- applications of quadratic shapes in geometry and analysis;
- concepts of conjugate and orthogonal transformation;
- applications of Euclidean metrics in geometry and analysis problems, various applications of the symmetric spectral problem;
- the ability to use the necessary literature to solve problems of increased difficulty (in the variable part of the course).

4. Content of the course (training module), structured by topics (sections), indicating the number of allocated academic hours and types of training sessions

4.1. The sections of the course (training module) and the complexity of the types of training sessions

№	Topic (section) of the course	Types of training sessions, including independent work			
		Lectures	Seminars	Laboratory practical	Independent work
1	Matrices and systems of linear equations	4	6		14
2	Linear space	6	4		14
3	Linear dependences in linear spaces	4	7		14
4	Nonlinear dependences in linear spaces	6	7		18
5	Eucledean space	6	6		18
6	Unitary space	4			12
AH in total		30	30		90
Exam preparation		30 AH.			
Total complexity		180 AH., credits in total 4			

4.2. Content of the course (training module), structured by topics (sections)

Semester: 3 (Fall)

1. Matrices and systems of linear equations

Solving systems of linear equations by the Kramer method. The rank of a matrix. Basic minor theorem. Theorem on the rank of the matrix.
Linear equation system. Kronecker-Capelli Theorem. Fundamental system of solutions and General solution of a homogeneous system of linear equations. The General solution of the inhomogeneous system. Gauss method. The Theorem OfFredholm.

2. Linear space

The axioms of a linear space. Linear dependence and linear independence of element systems in linear space. Dimension and basis. Subspaces and linear shells in linear space. Sum and intersection of subspaces. Direct sum. The formula for the dimension of the sum of subspaces. Derivation of the dimension formula of the sum of subspaces. Hyperplanes.
The expansion of the basis in a linear space. Coordinate representation of linear space elements and operations with them. Theorem on isomorphism. The coordinate form of the necessary and sufficient condition of the linear dependence of the elements.

Change of coordinates when changing the basis in linear space. Transition matrix and its properties. The coordinate form of the task subspaces and hyperplanes.

3. Linear dependences in linear spaces

Linear mappings and linear transformations of linear space. Operations on linear transformations. Inverse transformation. Linear space of linear maps. Algebra of linear transformations.

Linear mapping and linear transformation matrices for finite dimensional spaces. Operations on linear transformations in coordinate form. The change of the matrix of the linear display when changing bases. Isomorphism of the space of linear maps and the space of matrices.

Invariant subspaces of linear transformations. Eigenvectors and eigenvalues. Own subspaces. Linear independence of eigenvectors belonging to different eigenvectors.

Finding eigenvalues and eigenvectors of linear transformation of finite-dimensional linear space. Characteristic equation. Evaluation of the dimension of the private subspace. Diagonalizability conditions of the linear transformation matrix. Reduction of the matrix of linear transformation to a triangular form.

Linear form. Conjugate (dual) space. Biorthogonal basis. Secondary conjugate space.

4. Nonlinear dependences in linear spaces

Bilinear and quadratic forms. Their coordinate representation in a finite-dimensional linear space. Changing the matrices of bilinear and quadratic forms when changing the basis.

Reduction of the quadratic form to the canonical form by the Lagrange method. Inertia theorem for quadratic forms. Sign-definite quadratic forms. Sylvester's Test. Reduction of the quadratic form to the diagonal form by elementary transformations. FormulationofJordan'stheorem.

5. Eucledean space

Axiomatics of Euclidean space. The Cauchy-Schwarz Inequality. Triangle inequality. Gram matrix and its properties.

Finite-dimensional Euclidean space. Orthogonalization of the basis. Transition from one orthonormal basis to another. The orthogonal complement of the subspace.

Linear transformations of Euclidean space. Orthogonal projection on the subspace. Conjugate transformations, their properties. The coordinate form of the conjugation of the finite-dimensional Euclidean space transformation.

Self-conjugate transformations. Properties of their eigenvectors and eigenvalues. Existence of a basis from eigenvectors of a self-adjoint transformation.

Orthogonal transformations. Their properties are a Coordinate sign of orthogonality. Properties of orthogonal matrices. Polar decomposition of linear transformations of Euclidean space. Canonical form of the orthogonal transformation matrix. Singular decomposition.

Construction of an orthonormal basis in which the quadratic form has a diagonal form. Simultaneous reduction to the diagonal form of a pair of quadratic forms, one of which is sign-definite.

6. Unitary space

Unitary space and its axiomatics. Unitary and Hermitian matrices. Unitary and Hermitian transformations. Hermitian form. Properties of unitary and Hermitian transformations. Properties of Hermitian forms.

The concept of tensors. Basic tensor operations. Tensors in Euclidean space. Tensors in orthonormal basis.

5. Description of the material and technical facilities that are necessary for the implementation of the educational process of the course (training module)

Classroom equipped with multimedia projector, screen and microphone.

6. List of the main and additional literature, that is necessary for the course (training module) mastering

Main literature

Strang, G.
Linear algebra and its applications / G. Strange ; Massachusetts Institute of Technology .— 4th edition .— USA : Brooks/Cole : Cengage Learning, 2006 .— 488 p. - Index: p. 482-487. - ISBN 978-0-03-010567-8.

Additional literature

Мантуров, О. В.
Курс высшей математики [Текст] : Линейная алгебра. Аналитическая геометрия. Дифференциальные исчисление функций одной переменной : учебник для втузов / О. В. Мантуров, Н. М. Матвеев .— М. : Высшая школа, 1986 .— 480 с. : ил. - Библиогр.: с. 475. - Предм. указ.: с. 476-480. - 70 000 экз. - ISBN 5-06-000758-6 .

7. List of web resources that are necessary for the course (training module) mastering

http://www.math.mipt.ru
http://lib.mipt.ru

8. List of information technologies used for implementation of the educational process, including a list of software and information reference systems (if necessary)

The lectures use multimedia technologies, including presentations.

9. Guidelines for students to master the course

Given in the annually developed homework.

SUPPLEMENT

Assessment funds for course (training module)

major:	Biotechnology
specialization:	Biomedical Engineering/Биомедицинская инженерия Phystech School of Biological and Medical Physics Chair of Higher Mathematics
term:	2
qualification:	Bachelor

Semester, form of interim assessment: 3 (fall) - Exam

Authors:

D.V. Beklemishev, doctor of pedagogical sciences, full professor, professor

A.N. Burmistrov, candidate of physics and mathematical sciences, associate professor, associate professor

P.A. Kozhevnikov, candidate of physics and mathematical sciences, associate professor, associate professor

O.K. Podlipskiy, candidate of physics and mathematical sciences, associate professor, associate professor

I.A. Chubarov, candidate of physics and mathematical sciences, associate professor, associate professor

O.G. Podlipskaya, candidate of physics and mathematical sciences, assistant

1. Competencies formed during the process of studying the course

Code and the name of the competence	Competency indicators
UC-1 Search and identify, critically assess, and synthesize information, apply a systematic approach to problem-solving	UC-1.1 Analyze problems, highlight the stages of their solution, plan the actions required to solve them
	UC-1.2 Find, critically assess, and select information required for the task in hand
	UC-1.3 Consider various options for solving a problem, assess the advantages and disadvantages of each option
	UC-1.4 Make competent judgments and estimates supported by logic and reasoning
UC-6 Use time-management skills, apply principles of self-development and lifelong learning	UC-6.2 Plan independent activities in professional problem-solving; critically analyze the work performed; find creative ways to use relevant experience for self-development

2. Competency assessment indicators

As a result of studying the course the student should:

know:

- operations with matrices, systems of linear algebraic equations matrices and determinants;
- theorems on systems of linear Kronecker-Capelli and Fredholm equations, Kramer rule, General solution of a system of linear equations;
- basic definitions and theorems on linear spaces and subspaces, on linear maps of linear spaces;
- definitions and basic properties of eigenvectors, eigenvalues, characteristic polynomial;
- reduction of the quadratic form to the canonical form, the law of inertia, the Sylvester criterion;
- coordinate recording of the scalar product, the basic properties of self-adjoint transformations;
- fundamentals of the theory of linear spaces in the volume that provides the study of analytical mechanics, theoretical physics and optimal control methods.

be able to:

- to produce a matrix, finding inverse of a matrix, to compute determinants;
- find a numerical solution to a system of linear equations. find eigenvalues and eigenvectors of linear transformations, bring the quadratic form to the canonical form, find the orthonormal basis of the eigenvectors of the self-adjoint transformation;
- operate with elements and concepts of linear space, including the main types of dependencies: linear operators, bilinear and quadratic forms.

master:

- general concepts and definitions related to matrix algebra;
- geometric interpretation of systems of linear equations and their solutions;
- concepts of linear space, matrix notation of subspaces and maps;
- conduct about the use of spectral problems;
- applications of quadratic shapes in geometry and analysis;
- concepts of conjugate and orthogonal transformation;
- applications of Euclidean metrics in geometry and analysis problems, various applications of the symmetric spectral problem;
- the ability to use the necessary literature to solve problems of increased difficulty (in the variable part of the course).

3. List of typical control tasks used to evaluate knowledge and skills

Current control is carried out on the basis of a point-rating system (BRS) for evaluating knowledge in the discipline being studied. The BRS takes into account the students ' performance of a set of homework assignments and tests in accordance with the curriculum. Data on attendance and current academic performance are entered by teachers in special journals and recorded in the BRS.

Current control on the basis of homework is carried out during the academic semester in the terms set by the Educational Department, in accordance with the curriculum.

To pass the task, the student must provide a solution to the homework problem in writing, answer the questions of the teacher and write a test paper on the task, which checks the knowledge of concepts and statements on the topics of the task and the ability to solve problems.

You can't use other people's help, computers, or mobile phones during the test.

* A BRS is attached to the subject being studied.

4. Evaluation criteria

Certification in the discipline "Linear Algebra/Линейная алгебра" is carried out in the form of an exam.

The examination is conducted in accordance with the previously performed by the students in the control tasks.

Control tasks:

1. Prove that all minors of order k in some matrix are 0, then minors of higher orders are also 0.
2. Prove that if the columns of the matrix of a system of linear equations are linearly independent, then the system has at most one solution.
3. What is the direct sum of linear subspaces? Prove that the space of all functions defined on the interval [-a, a] is the direct sum of the subspace of even functions and the subspace of odd functions.
4. How are the multiplicity of the root of the characteristic polynomial of the linear transformation and the dimension of the corresponding proper subspace related? Give an example when they are different.
5. How to change the matrix of transition from one basis to another, if :

a) swap the i-th and j-th vectors of the first basis;

b) swap the i-th and j-th vectors of the second basis;

C) arrange the vectors of both bases in reverse order?

6. What kind of matrix has a linear transformation in the basis E1,..., EP, if E1,..., EP (k<n) form a basis in an invariant subspace of the space?
7. Prove that the determinant of the linear transformation matrix does not change when the basis changes. Is this true for a matrix of quadratic form? Prove that in the transition from one orthonormal basis in Euclidean space to another the determinant of the matrix of quadratic form does not change.
8. What values can take the determinant of the orthogonal matrix? Is it true that if the determinant of the matrix modulo is 1, then it is orthogonal?
9. What is a positive (negative) definite quadratic form? Formulate the Sylvester criterion for positive definite quadratic forms its analogue for negatively definite quadratic forms.
10. Formulate the properties of the eigenvalues of self-adjoint and orthogonal transformations. Prove that the eigenvectors of these transformations corresponding to different eigenvalues are orthogonal.

Examples of examination tickets:

Ticket 1

1. Prove that all minors of order k in some matrix are 0, then minors of higher orders are also 0.
2. How to change the matrix of transition from one basis to another, if :

a) swap the i-th and j-th vectors of the first basis;

b) swap the i-th and j-th vectors of the second basis;

Ticket 2

1. Prove that if the columns of the matrix of a system of linear equations are linearly independent, then the system has at most one solution.
2. Formulate the properties of the eigenvalues of self-adjoint and orthogonal transformations. Prove that the eigenvectors of these transformations corresponding to different eigenvalues are orthogonal.

Grade "excellent (10)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks;

Grade "excellent (9)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently found and corrected;

Grade "excellent (8)" is given to a student who has exhibited extensive and deep knowledge of the course and ability to apply skills when solving specific tasks, but he has made minor errors that were independently corrected after the instructions of an examiner;

Grade "good (7)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made minor mistakes when answering questions or solving problems;

Grade "good (6)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made rare mistakes when answering questions or solving problems;

Grade "good (5)" is given to a student who has a good command of the course and is able to apply skills when solving specific tasks, but has made mistakes when answering questions or solving problems;

Grade "satisfactory (4)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, but understands the subject well, is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "satisfactory (3)" is given to a student who has exhibited fragmented knowledge, has made inaccurate formulation of the basic concepts, has inconsistencies in understanding the course, but is able to apply the knowledge in standard situations and possesses skills necessary for the future study;

Grade "unsatisfactory (2)" is given to a student who does not possess knowledge of the essential concept of the course, has made gross mistakes in formulations of basic concepts and cannot use the knowledge in solving typical tasks;

Grade "unsatisfactory (1)" is given to a student who has exhibited total lack of knowledge of the course.

5. Methodological materials defining the procedures for the assessment of knowledge, skills, abilities and/or experience

During the oral examination, the student is given one hour (astronomical) to prepare. Poll student ticket on the oral exam should not exceed two hours.
During the examination, students can use the discipline program.

«СОГЛАСОВАНО»

Проректора по учебной работе и довузовской подготовке

_______________ А.А. Воронов

«____» _________________ 2020

Балльно-рейтинговая система оценки знаний студентов

Дисциплина: **Linear Algebra/Линейная алгебра**
2 курс, 3 семестр, экзамен

Кафедра: **высшей математики**

№	**Вид занятий**	**Сумма баллов**
1.	Посещение лекций	0–3
2.	Проверка теоретических знаний	0–3
3.	Контрольная работы, проводимые в классе	0–18
4.	Домашняя работа	0–6
5.	Итоговый контроль Экзамен (устный ответ)	0–70
	ИТОГО	**0–100**

*Если при учете этого вида работы итоговая сумма за работу в семестре превосходит 30 баллов, то считать ее равной 30 баллам.

Сумма баллов за устный ответ начисляется по формуле $N * 7$, где $N \geq 3$ — предварительная оценка за устный ответ по десятибалльной шкале. Если $N = 1, 2$, то итоговая оценка совпадает с N.

Соответствие оценок итоговой академической успеваемости балльно-рейтинговой системы.

Баллы БРС	**Оценки**	
93–100	10	отлично
86–92	9	
79–85	8	
72–78	7	хорошо
65–71	6	
58–64	5	
51–57	4	удовлетворительно
44–50	3	
30–43	2	неудовлетворительно
0–29	1	

Регламент принятия домашних заданий и проведения экзамена определяется «Положением о текущем контроле успеваемости и промежуточной аттестации студентов на кафедре высшей математики».

Зав.кафедрой _________________ Г. Е. Иванов